Book Review

V. S. Mikhalevich, A. M. Gupal, and V. I. Norkin, *Methods of Nonconvex Optimization* (in Russian), Nauka, Moscow, 1987, 280 pp. Price: 3 rubls 10 kop.

The book deals with the finite dimensional problems of nonconvex optimization and their numerical solution methods. The theory of extremum problem complexity states that nonconvex problems are an extraordinary challenge for the researchers (efforts to obtain their guaranteed solution grow exponentially with the increase of the dimension). In spite of the discouraging statement of complexity, the impotence of those problems to practice gives an impetus to the elaboration of numerical methods.

Two families of nonconvex functions are investigated in the book – the so-called generalized differentiable functions and the locally Lipschitz functions. These families are sufficiently general and cover the majority of practical demands. In particular, the functions under consideration may be nonsmooth. The nonsmooth functions are natural in many practical applications. They often appear in the theory of extremum problems as a result of using decomposition methods, exact nonsmooth penalties, duality and others. The classes of generalized differentiable functions and locally Lipschitz functions include convex and concave functions and their differences, and they are closed with respect to operations of maximum, minimum, superposition and mathematical expectation.

The authors introduce a notion of gradient for generalized differentiable functions, construct a calculus of such gradients, and study different properties of nonconvex problems in terms of these gradients. They extend the theory and algorithms of the sub-gradient convex optimization (including stochastic sub-gradient methods) to the optimization problems with the generalized differentiable functions.

A technique for optimization of Lipschitz functions is also proposed. Lipschitz function is approximated by the family of smooth functions. Only some estimates of these smooth function gradients are used in the solution procedure. The gradients of the smooth functions are approximated by the stochastic finite differences. For calculation of stochastic differences, the values of the original Lipschitz function are used. A similar approach is developed for the Lipschitz stochastic programming problems. Different generalizations of the classic stochastic approximation method are obtained for the solution of Lipschitz stochastic constrained programming problems.

In this book, attention mainly is paid to the local constrained optimization of generalized differentiable functions and locally Lipschitz functions, but the global optimization problem is also discussed. Three approaches to the global optimization are considered:

- *The combined algorithms of nonconvex nonsmooth optimization.* According to this approach, local methods are used for searching of local minimums and global optimization methods are used for going out of the attraction zone of the local minimum. The convergence conditions of the combined algorithms are investigated.
- *A generalization of the classic Piyavski methods for problems with general nonlinear constraints.* The auxiliary problems arising in this method are deduced either to concave minimization problems or to reverse convex programming problems.
- *A smoothing method for solving unconstrained global optimization problems.* The smoothing procedure deletes small local extremums of the original function.

Besides these three methods, the efficient local heavy ball-and-ravine-step methods are also considered in the book. The motion in these methods is made along the valleys of the minimized function. Since these methods have some inertia properties, they can rush through local minimums and thus may be used for the global optimization.

The various chapter headings are as follows: Introduction; 1. Elements of Nonconvex Analysis Theory; 2. Minimization of Lipschitz Functions without Computation of Gradients; 3. Generalized Gradient Descent Methods; 4. Non-Monotone Average Direction Methods; 5. Solution of Constrained Extremum Problems with Lipschitz Functions; 6. Random Lipschitz and Generalized Differentiable Functions; 7. Solution of Stochastic Extremum Problems; Literature Comments; and References.

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